

Class X Session 2025-26
Subject - Mathematics (Basic)
Sample Question Paper - 01

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

Section A

1. If the HCF of 360 and 64 is 8, then their LCM is: [1]
a) 2480 b) 2880
c) 512 d) 2780
2. If the prime factorisation of 2520 is $2^3 \times 3^a \times b \times 7$, then the value of $a + 2b$ is: [1]
a) 9 b) 10
c) 7 d) 12
3. Which of the following equations has 2 as a root? [1]
a) $x^2 + 3x - 12 = 0$ b) $x^2 - 4x + 5 = 0$
c) $2x^2 - 7x + 6 = 0$ d) $3x^2 - 6x - 2 = 0$
4. The value of k for which the equations $3x - y + 8 = 0$ and $6x + ky = -16$ represent coincident lines, is: [1]

a) $-\frac{1}{2}$

b) 2

c) $\frac{1}{2}$

d) -2

5. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then [1]

a) $k > 4$

b) $k < 4$

c) $k \leq 4$

d) $k \geq 4$

6. Points (6, 8), (3, 7), (-2, -2) and (1, -1) are joined to form a quadrilateral. What will be the structure of the quadrilateral? [1]

a) Rectangle

b) Square

c) Parallelogram

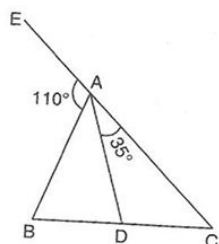
d) Rhombus

7. In a $\triangle ABC$, perpendicular AD from A on BC meets BC at D. If BD = 8 cm, DC = 2 cm and AD = 4 cm, then: [1]

a) $AC = 2 AB$

b) $\triangle ABC$ is equilateralc) $\triangle ABC$ is right - angled at A.d) $\triangle ABC$ is isosceles

8. In the adjoining figure if exterior $\angle EAB = 110^\circ$, $\angle CAD = 35^\circ$, AB = 5cm, AC = 7cm and BC = 3cm, then CD is equal to [1]



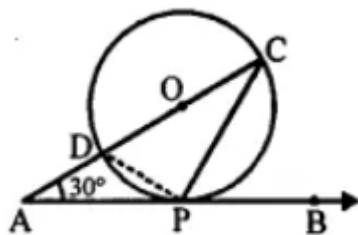
a) 2 cm.

b) 1.75 cm.

c) 1.9 cm.

d) 2.25 cm.

9. In the given figure, O is the centre of the circle. AB is the tangent to the circle at the point P. If $\angle PAO = 30^\circ$ then $\angle CPB + \angle ACP$ is equal to [1]



a) 150°

b) 60°

c) 120°

d) 90°

10. $\cos^2 30^\circ \cos^2 45^\circ + 4 \sec^2 60^\circ + \frac{1}{2} \cos^2 90^\circ - 2 \tan^2 60^\circ = ?$ [1]

a) $\frac{81}{8}$

b) $\frac{73}{8}$

c) $\frac{83}{8}$

d) $\frac{75}{8}$

11. The angle of elevation of the sun when the shadow of a pole 'h' metres high is $\frac{h}{\sqrt{3}}$ metres long is [1]

a) 15°

b) 30°

c) 45°

d) 60°

12. If $\sin \theta = \frac{a}{b}$, then $\sec \theta$ is equal to ($0 \leq \theta \leq 90^\circ$): [1]

 - $\frac{\sqrt{b^2-a^2}}{b}$
 - $\frac{\sqrt{b^2-a^2}}{a}$
 - $\frac{a}{\sqrt{b^2-a^2}}$
 - $\frac{b}{\sqrt{b^2-a^2}}$

13. The area of a sector of a circle with radius r, making an angle of x° at the centre is [1]

 - $\frac{x}{360} \times 2xr$
 - $\frac{x}{180} \times 2\pi$
 - $\frac{x}{360} \times \pi r^2$
 - $\frac{x}{100} xw^2$

14. Area of a quadrant of circle whose circumference is 22 cm is ($\pi = \frac{22}{7}$) [1]

 - 3.65 cm^2
 - 17.25 cm^2
 - 9.625 cm^2
 - 9.625 cm^3

15. A bag contains 50 balls of which 2x are red, 3x are white and 5x are blue. A ball is selected at random. The probability that it is not white is [1]

 - $\frac{7}{10}$
 - $\frac{7}{45}$
 - $\frac{3}{5}$
 - $\frac{2}{5}$

16. The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is: [1]

 - 22
 - 17
 - 27
 - 23

17. A circus tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent. [1]

 - 2096 m
 - 1947 m
 - 1800 m
 - 1996 m

18. The median of first 10 prime numbers is: [1]

 - 13
 - 11
 - 12.5
 - 12

19. **Assertion (A):** Distance of (-4, 3) from x-axis is 5 units. [1]
Reason (R): Distance of point (a, b) from x-axis is |b| (mod b) units.

 - Both A and R are true and R is the correct explanation of A.
 - Both A and R are true but R is not the correct explanation of A.
 - A is true but R is false.
 - A is false but R is true.

20. **Assertion (A):** H.C.F. of smallest prime and smallest composite is 2. [1]
Reason (R): Smallest prime is 2 and smallest composite is 4 so their H.C.F. is 2.

 - Both A and R are true and R is the correct explanation of A.
 - Both A and R are true but R is not the correct explanation of A.
 - A is true but R is false.
 - A is false but R is true.

Section B

21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the pair of linear equations **[2]**

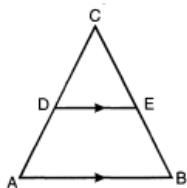


intersect at a point, are parallel or coincident: $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

22. In a $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D. If AB = 10 cm, AC = 6 cm and BC = 12 cm, find BD and DC. [2]

OR

In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE \parallel AB$.

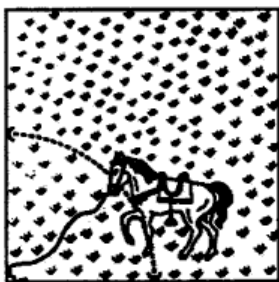


23. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. [2]
24. If $a \cos \theta - b \sin \theta = c$, prove that $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$ [2]
25. A car has two wipers which do not overlap. Each wiper has a blade of length 25cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades. [2]

OR

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find

- the area of that part of the field in which the horse can graze.
- the increase in the grazing area if the rope were 10 m long instead of 5 m (Use $\pi = 3.14$)



Section C

26. Show that $5 - \sqrt{3}$ is irrational. [3]
27. Find a quadratic polynomial whose sum and product of the zeroes are $-2\sqrt{3}$, -9 respectively. Also find the zeroes of the polynomial by factorisation. [3]
28. Solve the pair of linear equations $x + y = 14$ and $x - y = 4$ by substitution method. [3]

OR

Aditya is walking along the line joining points (1,4) and (0,6). Aditi is walking along the line joining points (3,4) and (1,0). Represent the graph and find the point where both cross each other.

29. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle a tangent is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of the triangle PCD. [3]
30. Evaluate: $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$ [3]

OR

Prove that:

$$\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$$

31. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25? [3]

Section D

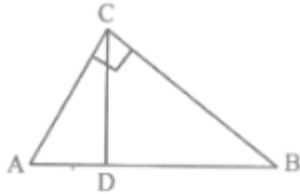


32. Find the value of m for which the quadratic equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$, $m \neq -1$ has equal roots. Hence find the roots of the equation. [5]

OR

Find all the values of k for which the quadratic equation $2x^2 + kx + 8 = 0$ has equal roots. Also, find the roots.

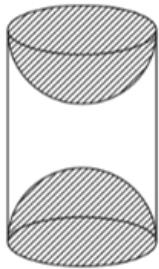
33. In the given figure, $\angle ACB = 90^\circ$ and $CD \perp AB$, Prove that $CD^2 = BD \times AD$. [5]



34. A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21} \text{ m}^3$ of air. [5]

OR

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 15 cm and its base is of radius 4.2 cm, then find the total surface area of the article.



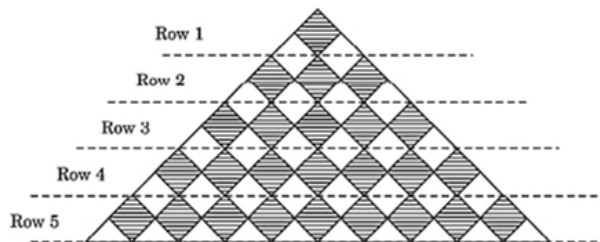
35. If the median of the distribution given below is 28.5, then find the values of x and y . [5]

Class Interval	frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Section E

36. Read the following text carefully and answer the questions that follow: [4]

A fashion designer is designing a fabric pattern. In each row, there are some shaded squares and unshaded triangles.



- Identify A.P. for the number of squares in each row. (1)
- Identify A.P. for the number of triangles in each row. (1)

iii. If each shaded square is of side 2 cm, then find the shaded area when 15 rows have been designed. (2)

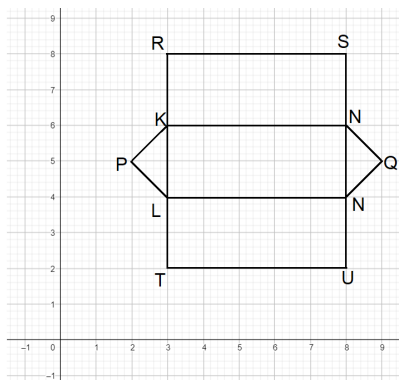
OR

Write a formula for finding total number of triangles in n number of rows. Hence, find S_{10} . (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

The camping alpine tent is usually made using high-quality canvas and it is waterproof. These alpine tents are mostly used in hilly areas, as the snow will not settle on the tent and make it damp. It is easy to layout and one need not use a manual to set it up. One alpine tent is shown in the figure given below, which has two triangular faces and three rectangular faces. Also, the image of canvas on graph paper is shown in the adjacent figure.



i. What is the distance of point Q from y-axis? (1)

ii. What are the coordinates of U? (1)

iii. What is the distance between the points P and Q? (2)

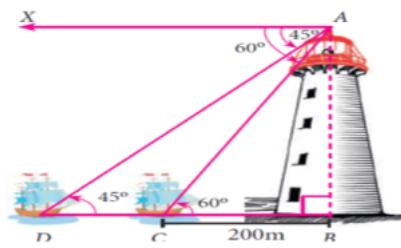
OR

What is the Perimeter of image of a rectangular face? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45° .



i. What is the approximate speed of the boat (in km/hr), assuming that it is sailing in still water? (1)

ii. How far is the boat when the angle is 45° ? (1)

iii. What is the height of tower? (2)

OR

As the boat moves away from the tower, angle of depression will decrease/increase? (2)

Solution

Section A

1.

(b) 2880

Explanation:

$$\text{LCM} = \frac{(\text{Number 1} \times \text{Number 2})}{\text{HCF}}$$

$$\text{LCM} = \frac{360 \times 64}{8}$$

$$\text{LCM} = \frac{23040}{8}$$

$$\text{LCM} = 2880$$

So, the LCM of 360 and 64 is 2880.

2.

(d) 12

Explanation:

$$2520 = 2^3 \times 3^2 \times 5 \times 7$$

on comparing

$$a = 2, b = 5$$

So,

$$a + 2b = 2 + 2 \times 5$$

$$= 12$$

3.

$$(c) 2x^2 - 7x + 6 = 0$$

Explanation:

$$\text{Given, } 2x^2 - 7x + 6 = 0$$

If 2 satisfies the above equation then 2 is a root.

$$\text{Now, } 2(2)^2 - 7(2) + 6 = 0$$

\therefore 2 is a root of this equation

4.

(d) -2

Explanation:

Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \dots (i)$$

Given lines are,

$$3x - y + 8 = 0$$

$$\text{and } 6x + ky + 16 = 0;$$

Comparing with the standard form, gives

$$a_1 = 3, b_1 = -1, c_1 = 8;$$

$$a_2 = 6, b_2 = k, c_2 = 16;$$

$$\text{and, from Eq. (i), } \frac{3}{6} = \frac{-1}{k} = \frac{8}{16}$$

$$\frac{-1}{k} = \frac{1}{2}$$

$$\text{So, } k = -2$$

5.

(b) $k < 4$



Explanation:

In the equation $x^2 + 4x + k = 0$

$a = 1, b = 4, c = k$

$$D = b^2 - 4ac = (4)^2 - 4 \times 1 \times k = 16 - 4k$$

Roots are real and distinct

$$D > 0$$

$$\Rightarrow 16 - 4k > 0$$

$$\Rightarrow 16 > 4k$$

$$\Rightarrow 4 > k$$

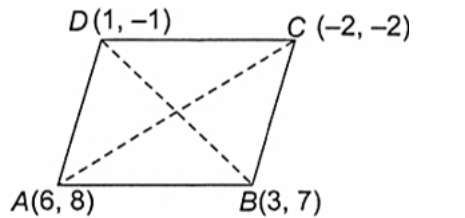
$$\Rightarrow k < 4$$

6.

(c) Parallelogram

Explanation:

Let the points be A(6, 8), B(3, 7), C(-2, -2) and D(1, -1).



$$\text{Now, } AB = \sqrt{(3-6)^2 + (7-8)^2} = \sqrt{10}$$

$$BC = \sqrt{(-2-3)^2 + (-2-7)^2} = \sqrt{106}$$

$$CD = \sqrt{(1+2)^2 + (-1+2)^2} = \sqrt{10}$$

$$DA = \sqrt{(6-1)^2 + (8+1)^2} = \sqrt{106}$$

$$\text{Also, } AC = \sqrt{(-8)^2 + (-10)^2} = \sqrt{64 + 100} = \sqrt{164}$$

$$BD = \sqrt{(-2)^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68}$$

Since, $AB = DC$ and $BC = DA$ and $AC \neq BD$.

\therefore ABCD is a parallelogram.

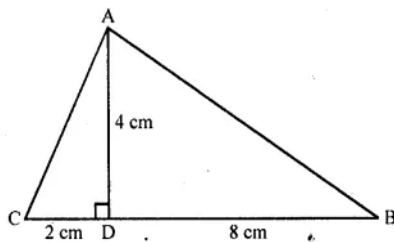
7.

(c) $\triangle ABC$ is right - angled at A.

Explanation:

In $\triangle ABC$, $AD \perp BC$

$BD = 8$ CM, $DC = 2$ CM, $AD = 4$ CM



In right $\triangle ACD$,

$$AC^2 = AD^2 + CD^2 \text{ (Pythagoras Theorem)}$$

$$= (4)^2 + (2)^2 = 16 + 4 = 20$$

and in right $\triangle ABD$,

$$AB^2 = AD^2 + DB^2$$

$$= (4)^2 + (8)^2 = 16 + 64 = 80$$

$$\text{and } BC^2 = (BD + DC)^2 = (8 + 2)^2 = (10)^2 = 100$$

$$AB^2 + AC^2 = 80 + 20 = 100 = BC^2$$

$\triangle ABC$ is a right triangle whose $\angle A = 90^\circ$

8.

(b) 1.75 cm.

Explanation:

Here, $\angle BAD = 180^\circ - (\angle EAB + \angle ADC) = 180^\circ - 110^\circ - 35^\circ = 35^\circ$

Since, AD bisects $\angle A$.

$\therefore \frac{AB}{AC} = \frac{BD}{CD}$ [Internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle]

$$\Rightarrow \frac{5}{7} = \frac{3-CD}{CD}$$

$$\Rightarrow 5CD = 21 - 7CD \Rightarrow 5CD + 7CD = 21$$

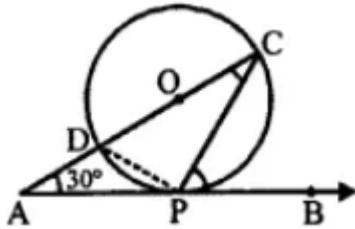
$$\Rightarrow 12CD = 21 \Rightarrow CD = 1.75 \text{ cm}$$

9.

(d) 90°

Explanation:

In the given figure, O is the centre of the circle. AB is tangent to the circle at P.



$$\angle PAO = 30^\circ$$

$$\angle CPB + \angle ACP = ?$$

$$\angle CPD = 90^\circ \text{ (Angle in a semi circle)}$$

$$\angle DPA + \angle CPB = 90^\circ$$

$$\text{But } \angle DPA = \angle ACP \text{ (Angles in alternate segment)}$$

$$\angle CPB + \angle ACP = 90^\circ$$

10.

(c) $\frac{83}{8}$

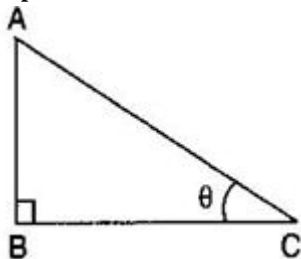
Explanation:

$$\begin{aligned} & \cos^2 30^\circ \cos^2 45^\circ + 4 \sec^2 60^\circ + \frac{1}{2} \cos^2 90^\circ - 2 \tan^2 60^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + (4 \times 2^2) + \left(\frac{1}{2} \times 0^2\right) - 2 \times (\sqrt{3})^2 \\ &= \left(\frac{3}{4} \times \frac{1}{2}\right) + 16 + 0 - 6 = \frac{3}{8} + 10 = \frac{83}{8} \end{aligned}$$

11.

(d) 60°

Explanation:



Given: Height of the pole = $AB = h$ meters And the length of the shadow of the pole = $BC = \frac{h}{\sqrt{3}}$ meters $\therefore \tan \theta = \frac{h}{\frac{h}{\sqrt{3}}}$

$$\Rightarrow \tan \theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

12.

(d) $\frac{b}{\sqrt{b^2-a^2}}$

Explanation:

$$\sin \theta = \frac{a}{b}$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$= \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{\frac{b^2 - a^2}{b^2}}$$

Now,

$$\sec \theta = \frac{1}{\cos \theta}$$

$$= \sqrt{\frac{b^2}{b^2 - a^2}}$$

$$= \frac{b}{\sqrt{b^2 - a^2}}$$

13.

(c) $\frac{x}{360} \times \pi r^2$

Explanation:

Area of a sector of a circle with radius r and making an angle of x°

at the centre = $\frac{x}{360} \times \pi r^2$

14.

(c) 9.625 cm^2

Explanation:

We have

$$C = 22 \text{ cm}$$

$$\Rightarrow 2\pi r = 22 \text{ cm}$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\therefore r = 3.5 \text{ cm}$$

Now,

$$\text{Area of quadrant of a circle} = \frac{\pi r^2}{4}$$

$$= \frac{22}{7} \times \frac{(3.5)^2}{4}$$

$$= \frac{22 \times 12.25}{28}$$

$$= 9.625 \text{ cm}^2$$

15. (a) $\frac{7}{10}$

Explanation:

Here,

$$2x + 3x + 5x = 50$$

$$\Rightarrow 10x = 50$$

$$\Rightarrow x = 5$$

$$\text{Number of red balls} = 2 \times 5 = 10$$

$$\text{Number of white balls} = 3 \times 5 = 15$$

$$\text{Number of blue balls} = 5 \times 5 = 25$$

$$\text{Now, Number of possible outcomes} = 25 + 10 = 35$$

$$\text{And Number of total outcomes} = 50$$

$$\therefore \text{Required Probability} = \frac{35}{50} = \frac{7}{10}$$

16.

(c) 27

Explanation:

We know that

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$



$$3 \times 23 = \text{Mode} + 2 \times 21$$

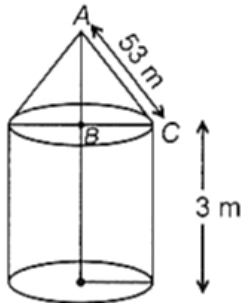
$$69 - 42 = \text{Mode}$$

$$\text{Mode} = 27$$

17.

(b) 1947 m

Explanation:



For cylindrical part,

$$\text{Radius (r)} = \frac{105}{2} \text{ m}$$

$$\text{Height (h)} = 3 \text{ m}$$

For conical part,

$$\text{Slant height (l)} = 53 \text{ m}$$

$$\text{Radius (r)} = \frac{105}{2} \text{ m}$$

$$\therefore \text{Total curved surface area of tent} = 2\pi rh + \pi rl$$

$$= \pi r(2h + l) = \frac{22}{7} \times \frac{105}{2} \times (6 + 53) = (11 \times 15 \times 59) \text{ m}^2$$

Hence, length of canvas

$$= \frac{\text{Total curved surface area of tent}}{\text{Width of cloth}} = \frac{11 \times 15 \times 59}{5} = 1947 \text{ m}$$

18.

(d) 12

Explanation:

The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Here $n = 10$, which is even number.

$$\therefore \text{Median} = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term} \right]$$

$$= \frac{1}{2} [5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}] = \frac{1}{2} [11 + 13]$$

$$= \frac{1}{2} \times 25$$

$$= 12$$

19.

(d) A is false but R is true.

Explanation:

A is false but R is true.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

Smallest prime is 2 and smallest composite is 4 so H.C.F. of 2 and 4 is 4.

Section B

21. Given equations are

$$9x + 3y + 12 = 0$$

$$18x + 6y + 24 = 0$$

$$\text{Comparing equation } 9x + 3y + 12 = 0 \text{ with } a_1x + b_1y + c_1 = 0$$

$$\text{and } 18x + 6y + 24 = 0 \text{ with}$$

$$a_2x + b_2y + c_2 = 0,$$

We get, $a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18, b_2 = 6, c_2 = 24$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Hence, lines are coincident.

22. It is given that $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm

In $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D

We have to find BD and DC

Since AD is $\angle A$ bisector

$$\text{So } \frac{AC}{AB} = \frac{DC}{BD}$$

Let $BD = x$ cm

$$\text{Then, } \frac{6}{10} = \frac{12-x}{x}$$

$$\Rightarrow 6x = 120 - 10x$$

$$\Rightarrow 16x = 120$$

$$\Rightarrow x = \frac{120}{16}$$

$$\Rightarrow x = 7.5$$

Now

$$DC = 12 - BD$$

$$= 12 - 7.5$$

$$= 4.5$$

Hence, $BD = 7.5$ cm and $DC = 4.5$ cm

OR

In $\triangle CAB$, $\angle A = \angle B$ (Given)

$\therefore AC = CB$ (By isosceles triangle property)

But, $AD = BE$ (Given).....(i)

$$\Rightarrow AC - CD = CB - BE$$

$$\therefore CD = CE \text{(ii)}$$

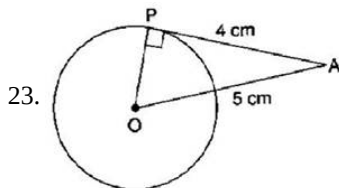
Dividing equation (ii) by (i),

$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

$$DE \parallel AB.$$

\therefore If $\angle A = \angle B$ and $AD = BE$ then, $DE \parallel AB$.



23.

We know that the tangent at any point of a circle is \perp to the radius through the point of contact.

$$\therefore \angle OPA = 90^\circ$$

$$\therefore OA^2 = OP^2 + AP^2 \text{ [By Pythagoras theorem]}$$

$$\Rightarrow (5)^2 = (OP)^2 + (4)^2$$

$$\Rightarrow 25 = (OP)^2 + 16$$

$$\Rightarrow OP^2 = 9$$

$$\Rightarrow OP = 3 \text{ cm}$$

24. Given, $a \cos \theta - b \sin \theta = c$

Squaring on both sides

$$(a \cos \theta - b \sin \theta)^2 = c^2$$

By Adding $(a \sin \theta + b \cos \theta)^2$ on both sides, we get

$$(a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2 = c^2 + (a \sin \theta + b \cos \theta)^2$$

$$(a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cos \theta) + (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2 + (a \sin \theta + b \cos \theta)^2$$

$$a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = c^2 + (a \sin \theta + b \cos \theta)^2$$

$$a^2 + b^2 = c^2 + (a \sin \theta + b \cos \theta)^2$$

$$\Rightarrow (a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$$

25. Radius of each wiper = 25cm, Angle = 115°

$$\therefore \theta = 115^\circ$$

Total area cleaned at each sweep of the blades

$$= 2 \left[\frac{115}{360} \times \frac{22}{7} \times 25 \times 25 \right] \left(\because \text{Area} = \frac{\theta}{360} \pi r^2 \right)$$

$$= \frac{230 \times 22 \times 5 \times 25}{72 \times 7}$$

$$= \frac{230 \times 11 \times 125}{36 \times 7}$$

$$= \frac{115 \times 11 \times 125}{18 \times 7}$$

$$= \frac{158125}{126} \text{ cm}^2$$

$$= 1254.96 \text{ cm}^2$$

OR

i. The area of that part of the field in which the horse can graze if the length of the rope is 5cm

$$= \frac{1}{4} \pi r^2 = \frac{1}{4} \times 3.14 \times (5)^2 = \frac{1}{4} \times 78.5 = 19.625 \text{ m}^2$$

ii. The area of that part of the field in which the horse can graze if the length of the rope is 10 m

$$= \frac{1}{4} \pi r^2 = \frac{1}{4} \times 3.14 \times (10)^2 = 78.5 \text{ m}^2$$

\therefore The increase in the grazing area

$$= 78.5 - 19.625 = 58.875 \text{ m}^2$$

Section C

26. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime numbers a and b ($b \neq 0$) such that $5 - \sqrt{3} = \frac{a}{b}$

$$\text{Therefore, } 5 - \frac{a}{b} = \sqrt{3}$$

$$\text{Rearranging this equation, we get } \sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

27. Here, $\alpha + \beta = -2\sqrt{3}$ and $\alpha\beta = -9$

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta \text{ [Formula]}$$

$$= x^2 - (-2\sqrt{3})x + (-9)$$

$$\Rightarrow f(x) = x^2 + 2\sqrt{3}x - 9$$

For zeroes of polynomial f(x), $f(x) = 0$

$$\Rightarrow x^2 + 2\sqrt{3}x - 9 = 0$$

$$\Rightarrow x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$$

$$\Rightarrow x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$$

$$\Rightarrow (x + 3\sqrt{3})(x - \sqrt{3}) = 0$$

$$\Rightarrow x + 3\sqrt{3} = 0 \text{ or } (x - \sqrt{3}) = 0$$

$$\Rightarrow x = -3\sqrt{3} \text{ or } x = \sqrt{3}$$

$$\therefore \alpha = -3\sqrt{3} \text{ and } \beta = \sqrt{3}$$

Hence the polynomial is $x^2 + 2\sqrt{3}x - 9$ and its zeros are $-3\sqrt{3}$ and $\sqrt{3}$.

28. $x + y = 14$; $x - y = 4$

the given pair of linear equations is

$$x + y = 14 \dots\dots\dots(1)$$

$$x - y = 4 \dots\dots\dots(2)$$

From equation(1),

$$y = 14 - x \dots\dots\dots(3)$$

Substitute this value of y in equation(2), we get

$$x - (14 - x) = 4$$

$$\Rightarrow x - 14 + x = 4$$

$$\Rightarrow 2x - 14 = 4$$

$$\Rightarrow 2x = 4 + 14$$



$$\Rightarrow 2x = 18$$

$$\Rightarrow x = \frac{18}{2} = 9$$

Substituting this value of x in equation (3), we get $y = 14 - 9 = 5$

Therefore, the solution is $x = 9, y = 5$

verification: Substituting $x = 9$ and $y = 5$, we find that both the equations (1) and (2) are satisfied as shown below:

$$x + y = 9 + 5 = 14$$

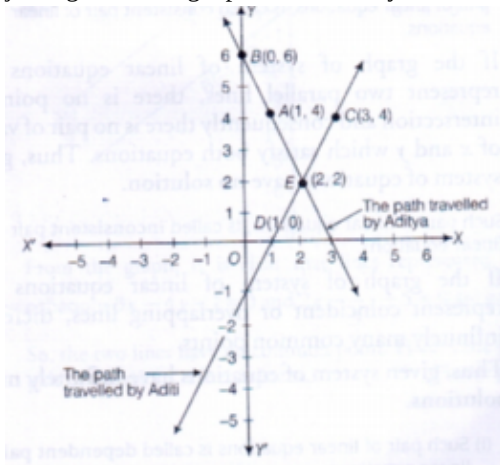
$$x - y = 9 - 5 = 4$$

This verifies the solution.

OR

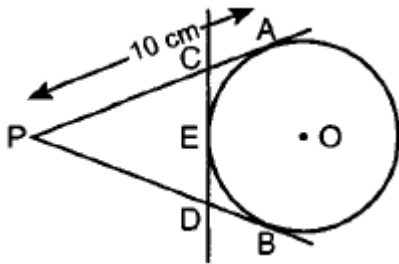
Let the given points be A(1,4), B(0,6), C(3,4) and D(1,0).

On plotting points A and B and joining them, we get the path travelled by Aditya. Similarly, on plotting points C and D and joining them, we get path travelled by Aditi.



It is clear from the graph that both of them cross each other at point E(2,2).

29. Given,



$$PA = 10 \text{ cm.}$$

$PA = PB$ [If P is external point] ... (i) [From an external point tangents drawn to a circle are equal in length]

If C is external point, then $CA = CE$

If D is external point, then

$$DB = DE \text{ ... (ii)}$$

Perimeter of triangle $\triangle PCD$

$$= PC + CD + PD$$

$$= PC + CE + ED + PD$$

$$= PC + CA + DB + PD$$

$$= PA + PB$$

$$= PA + PA$$

$$= 2PA$$

$$= 2 \times 10 = 20 \text{ cm [From (i)]}$$

$$30. \text{ We have } \frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

after putting values, we get

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$\begin{aligned}
&= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \text{ Rationalise it, we get} \\
&= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} \\
&= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2} \\
&= \frac{27+16-24\sqrt{3}}{27-16} \\
&= \frac{43-24\sqrt{3}}{11}
\end{aligned}$$

OR

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} \\
\Rightarrow \text{LHS} &= \frac{(\tan \theta + \sec \theta) - 1}{(\tan \theta - \sec \theta) + 1} \\
\Rightarrow \text{LHS} &= \frac{(\sec \theta + \tan \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1} \quad [\because \sec^2 \theta - \tan^2 \theta = 1] \\
\Rightarrow \text{LHS} &= \frac{(\sec \theta + \tan \theta) - (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{\tan \theta - \sec \theta + 1} \\
\Rightarrow \text{LHS} &= \frac{(\sec \theta + \tan \theta)[1 - (\sec \theta - \tan \theta)]}{\tan \theta - \sec \theta + 1} \\
\Rightarrow \text{LHS} &= \frac{(\sec \theta + \tan \theta)(1 - \sec \theta + \tan \theta)}{(\tan \theta - \sec \theta + 1)} \\
\Rightarrow \text{LHS} &= \frac{(\sec \theta + \tan \theta)(\tan \theta - \sec \theta + 1)}{(\tan \theta - \sec \theta + 1)} \\
\Rightarrow \text{LHS} &= \sec \theta + \tan \theta = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = \frac{1 + \sin \theta}{\cos \theta} = \text{RHS}
\end{aligned}$$

31. The person having higher probability of getting the number 25 has the better chance.

When a pair of dice is thrown, there are 36 elementary events which are as follows:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
 (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
 (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
 (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
 (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
 (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Therefore, the product of numbers on two dice can take values 1, 2, 3, ..., 36.

We observe that the product of two numbers on two dice will be 25 if both the dice show number 5. Therefore, there is only one elementary event, viz., (5, 5), which is favourable for getting 25.

$$p_1 = \text{Probability that Peter throws 25} = \frac{1}{36}$$

Rina throws a die on which she can get any one of the six numbers 1, 2, 3, 4, 5, 6 as an outcome. If she gets number 5 on the upper face of the die thrown, then the square of the number is 25.

$$p_2 = \text{Probability that the square of number obtained is 25} = \frac{1}{6}$$

Therefore, $p_2 > p_1$. Therefore, Rina has better chance to get the number 25.

Section D

32. In equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$

$$A = m+1, B = -6(m+1), C = 3(m+9)$$

$$\text{For equal roots, } D = B^2 - 4AC = 0$$

$$36(m+1)^2 - 4(m+1) \times 3(m+9) = 0$$

$$\Rightarrow 3(m^2 + 2m + 1) - (m+1)(m+9) = 0$$

$$\Rightarrow 2m^2 - 4m - 6 = 0$$

$$\Rightarrow m^2 - 2m - 3 = 0$$

$$\Rightarrow m^2 - 3m + m - 3 = 0$$

$$\Rightarrow m(m-3) + 1(m-3) = 0$$

$$\Rightarrow (m-3)(m+1) = 0$$

$$\therefore m = -1, 3$$

Neglecting $m \neq -1$

$$\therefore m = 3$$

$$\therefore \text{the equation becomes } 4y^2 - 24y + 36 = 0$$

$$\Rightarrow y^2 - 6y + 9 = 0$$

$$\Rightarrow (y-3)(y-3) = 0$$



$$\Rightarrow (y - 3) = 0 \quad \text{and} \quad (y - 3) = 0$$

\therefore roots are $y = 3, 3$

OR

For equal roots $k^2 - 64 = 0$

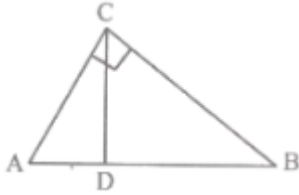
$$\Rightarrow k = \pm 8$$

Equations are $2x^2 + 8x + 8 = 0$ and $2x^2 - 8x + 8 = 0$

$$\Rightarrow 2(x + 2)^2 = 0 \quad \text{or} \quad 2(x - 2)^2 = 0$$

$$\Rightarrow x = -2 \quad \text{or} \quad x = 2$$

33. Given: $\angle ACB = 90^\circ$ and $CD \perp AB$



In $\triangle ACB$ and $\triangle ADC$

$$\angle ACB \cong \angle ADC \quad (\because \text{each } 90^\circ)$$

$$\angle A \cong \angle A \quad (\because \text{common angle})$$

$\triangle ACB \sim \triangle ADC$ (AA similarity)

$$\Rightarrow \frac{AC}{BC} = \frac{AD}{CD} \dots (i)$$

In $\triangle ACB$ and $\triangle CDB$

$$\angle ACB \cong \angle CDB \quad (\because \text{each } 90^\circ)$$

$$\angle B \cong \angle B \quad (\because \text{common angle})$$

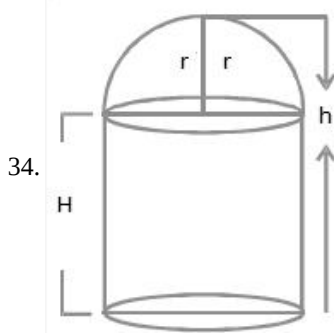
So, $\triangle ACB \sim \triangle CDB$ (AA similarity)

$$\Rightarrow \frac{AC}{BC} = \frac{CD}{BD} \dots (ii)$$

Using equations (i) and (ii),

$$\frac{AC}{BC} = \frac{CD}{BD}$$

$$\Rightarrow CD^2 = AD \times BD$$



34.

Let the radius of the hemispherical dome be r and the total height of the building be h .

Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height

$$2r = \frac{2}{3}h$$

$$\Rightarrow r = \frac{h}{3}$$

Let H be the height of the cylindrical position.

$$\Rightarrow H = h - r = h - \frac{h}{3} = \frac{2h}{3}$$

Volume of air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$$\Rightarrow 67\frac{1}{21} = \frac{2}{3}\pi r^3 + \pi r^2 H$$

$$\Rightarrow \frac{1408}{21} = \pi r^2 \left(\frac{2}{3}r + H \right)$$

$$\Rightarrow \frac{1408}{21} = \frac{22}{7} \times \left(\frac{h}{3} \right)^2 \left(\frac{2}{3} \times \frac{h}{3} + \frac{2h}{3} \right)$$

$$\Rightarrow \frac{1408 \times 7}{22 \times 21} = \frac{h^2}{9} \times \left(\frac{2h}{9} + \frac{2h}{3} \right)$$

$$\Rightarrow \frac{64}{3} = \frac{h^2}{9} \times \left(\frac{8h}{9} \right)$$

$$\Rightarrow \frac{64 \times 9 \times 9}{3 \times 8} = h^3$$

$$\Rightarrow h^3 = 8 \times 27$$

$$\Rightarrow h = 6$$

Thus, the height of the building is 6 m.

OR

Height of cylinder = 15 cm

Radius of cylinder = Radius of hemisphere = 4.2 cm

Total surface area = CSA of cylinder + CSA of 2 hemispheres

$$= 2\pi rh + 4\pi r^2$$

$$= 2 \times \frac{22}{7} \times 4.2 \times (15 + 2 \times 4.2)$$

$$= 2 \times \frac{22}{7} \times 4.2 \times 23.4 = 617.76 \text{ cm}^2$$

35.

Monthly Consumption	Number of consumers (f_i)	Cumulative Frequency
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
Total	$\sum f_i = n = 60$	

Here, $\sum f_i = n = 60$, then $\frac{n}{2} = \frac{60}{2} = 30$, also, median of the distribution is 28.5, which lies in interval 20 – 30.

\therefore Median class = 20 – 30

So, $l = 20$, $n = 60$, $f = 20$, $cf = 5 + x$ and $h = 10$

$$\therefore 45 + x + y = 60$$

$$\Rightarrow x + y = 15 \dots\dots\dots(i)$$

$$\text{Now, Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$$

$$\Rightarrow 28.5 = 20 + \left[\frac{30 - (5+x)}{20} \right] \times 10$$

$$\Rightarrow 28.5 = 20 + \frac{30 - 5 - x}{2}$$

$$\Rightarrow 28.5 = \frac{40 + 25 - x}{2}$$

$$\Rightarrow 57.0 = 65 - x$$

$$\Rightarrow x = 65 - 57 = 8$$

$$\Rightarrow x = 8$$

Putting the value of x in eq. (i), we get,

$$8 + y = 15$$

$$\Rightarrow y = 7$$

Hence the value of x and y are 8 and 7 respectively.

Section E

36. i. A.P. for the number of squares in each row is 1, 3, 5, 7, 9 ...

ii. A.P. for the number of triangles in each row is 2, 6, 10, 14 ...

iii. Area of each square = $2 \times 2 = 4 \text{ cm}^2$

$$\text{Number of squares in 15 rows} = \frac{15}{2}(2 + 14 \times 2) = 225$$

$$\text{Shaded area} = 225 \times 4 = 900 \text{ cm}^2$$

OR

$$S_n = \frac{n}{2}[4 + (n - 1)4] = 2n^2$$

$$\therefore S_{10} = 2 \times 10^2 = 200$$

37. i. Coordinates of Q are (9, 5).

\therefore Distance of point Q from y-axis = 9 units

ii. Coordinates of point U are (8, 2).

iii. We have, P(2, 5) and Q (9, 5)

$$\therefore PQ = \sqrt{(2-9)^2 + (5-5)^2} = \sqrt{49+0} = 7 \text{ units}$$

OR

Length of TU = 5 units and of TL = 2 units

$$\therefore \text{Perimeter of image of a rectangular face} = 2(5+2) = 14 \text{ units}$$

38. i. In $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{200}$$

$$AB = 200\sqrt{3}$$

Now, In $\triangle ABD$

$$\tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{200\sqrt{3}}{BD}$$

$$BD = 200\sqrt{3}$$

$$\therefore CD = BD - BC$$

$$= 200\sqrt{3} - 200$$

$$= 200(\sqrt{3} - 1)$$

$$= 200 \times (1.732 - 1)$$

$$= 200 \times 0.732$$

$$= 146.4 \text{ m}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{146.4}{10}$$

$$= 14.64 \text{ m/s}$$

Now,

$$\text{speed} = 14.64 \times \frac{18}{5} \text{ km/hr}$$

$$= 52.7$$

$$\approx 53 \text{ km/hr}$$

ii. In $\triangle ABD$

$$\tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{200\sqrt{3}}{BD}$$

$$BD = 200\sqrt{3} \text{ m}$$

$$\therefore CD = 200\sqrt{3} - 200$$

$$= 200(\sqrt{3} - 1)$$

$$= 200(1.732 - 1)$$

$$= 200 \times 0.732$$

$$= 146.4$$

$$\approx 147 \text{ m}$$

\therefore boat is at a distance of 147 m from its actual position.

iii. In $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{200}$$

$$AB = 200\sqrt{3} \text{ m}$$

Hence, height of tower = $200\sqrt{3} \text{ m}$

OR

As boat moves away from the tower angle of depression decreases.